# Research Proposal

*Implementing Grover's Diffusion Operator in A Single Photon, 2-Qubit Spatial Mode System*

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## Contents

# 1 Introduction:

We investigate the enhancement of communications protocols by treating the spatial modes of a single polarization-encoded photon as two separate qubits. In effect, we can recreate traditional 2-photon operations in an individual photon, improving the scability of quantum devices. To demonstrate the robustness of our approach, we will create path entanglement between the spatial mode qubits in the polarization basis. Experimentally, we will leverage a single photon source as the input, combined with beam splitters (BS), waveplates (WP), and polarizing beam splitters (PBS). To test for entanglement, we consider extending the Hong-Ou-Mandel effect to describe the bunching of two spatial modes at each of the four detectors. The second part of project will be in constructing the Grover's diffusion operator. Using our 2-qubit single photon system and our optically implemented controlled-NOT gate, we will use another set of waveplates to appropriately encode the polarization of the input states. Then at the detectors, we will observe the output states of the spatial modes to determine whether the amplitude has successfully been rotated. Note: in this case, we do not expect to see Hong-Ou-Mandel interference, as there will be no violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality for the operator.

## 1.1 Intellectual Merit

One of the primary contributions of our work is the exploration of producing entanglement within a photon using its spatial modes and encoded polarization states. By treating the spatial modes of the photon as two separate qubits, we can effectively replicate traditional 2-photon processes in one photon. This introduces an interesting theoretical approach of modifying the 1 qubit per photon ratio. Traditional 2:1 approaches, such as Spontaneous Parametric Down-Conversion (SPDC), generate entangled photon pairs probabilistically, often resulting in inefficiencies and high noise levels. By leveraging a linear approach with a single photon source combined with beam splitters and waveplates to achieve deterministic entanglement, we can potentially increase the efficiency and scalability of quantum communication systems.

## 1.2 Broader Impacts

The broader impact of this work lies in the construction of the diffusion operator. This operator, which rotates the amplitudes of the quantum states over their average, is integral to a bigger algorithm known as Grover's algorithm. Grover's algorithm essentially performs unstructured search for a given quantum state in $O(\sqrt{N})$ complexity. This is one of the first quantum algorithms proven to have an advantage over classical algorithms — classical binary search has a time complexity of $O(\log N)$. From a broader perspective, its primary application is in sifting through unsorted databases, a technique that will be incredibly useful in the age of Big Data. This can be extended to quantum machine learning (QML) algorithms which rely on pattern finding and recognition. By creating a 2-qubit system from a single photon, we can effectively scale Grover's algorithm without involving more physical systems. Furthermore, we aim to advance the field of photonics as a whole. One of the major challenges is gate implementation, as such protocols sometimes rely on probabilistic, non-linear operations. By introducing a fully linear approach in our circuit, we avoid low conversion efficiencies and spectral limitations associated with these nonlinear processes like spontaneous parametric down conversion (SPDC).

# 2 Specific Project Goals and Measurements:

1. **Entanglement Between Spatial Modes**
   First, we will test for indistinguishability, which serves as a prerequisite for entanglement in certain photonics protocols. To accomplish this, we consider extending the Hong-Ou-Mandel effect to spatial modes, as opposed to actual photons. In other words, if we consistently observe spatial mode bunching at each of the four detectors, we can reasonably conclude the modes are indistinguishable. Spatial modes bunch together into a single photon, meaning each photon coincidence at the detector indicates bunched spatial modes. Thus, we can directly measure this effect, as equal photon coincidences at the detectors will indicate successful indistinguishability. For further precision, we can establish an EMCCD or sCMOS camera system to analyze the spatial distribution of the photon coincidences. If the qubits are indistinguishable, we expect to see single spot corresponding to the bunched modes, whereas distinguishable qubits would have spatially separated spots. We can observe how this effect changes by rotating the HWP in increments of $\pi/4$ radians. To definitively prove entanglement, we will computationally test for the Clauser-Horne-Shimony-Holt inequality violation by parameterizing the circuit in Qiskit and using the calculated observable to visualize if it falls above and below Tsierlson's bounds.

2. **Diffusion Operator With Spatial Modes**
   Using the photon coincidences and camera-produced spatial distribution framework from the previous goal, we can analyze if the qubits are *distinguishable*. This occurs when the photon coincidences are unequal at the detectors. In other words, at some detectors we observe spatial mode bunching, while at other detectors it is less pronounced. This is caused by distinguishability in the modes, which we can further confirm by the photographs. This time, we expect to see two spatially separated dots. To be more specific, we expect one detector in particular to have a significantly higher number of photon coincidences, indicating that the marked state had its amplitude successfully amplified (as expected by inverting over the mean). When we remove the marking of the state (which we can accomplish by a phase shift rather than the full oracle), we do not expect to see that spike in coincidences at one particular detector.

# 3 Specific Responsibilities of Group Members:

All work – experimental and computational – will be handled by me.

# 4 Potential Pitfalls and Alternative Strategies:

1. Confirming indistinguishability

   Potential Challenge: Since we are aiming to optically produce entanglement between the spatial modes using a variety of components, we must ensure precise alignment of the beams. Misalignment is common due to a manual set-up and can end up severely skewing the results by preventing the modes from perfectly overlapping at the beamsplitter.

   Solution: We will consider using single mode fibers or waveguides to ensure no crosstalk between modes. Essentially, this will keep the beams separated at $90°$ for as long as they need to be before being recombined using mirrors. If this is not enough, apply a predefined machine learning model to measure the angle of error and use a spatial light modulator to correct for this.

   Potential Challenge: Once we ensure equal photon coincidences at each detector, we hope to visualize the bunching of spatial modes using an EMCCD of sCMOS camera. One specific challenge may be having a photon flux too low that it cannot be captured by the camera, which is a consequence of single photon systems.

   Solution: To address this, we can use a high-gain EMCCD and use long enough exposure times to accomodate for the sensitivity. Another challenge is noise from the readout process, which is inevitable with highly sensitive imaging systems. Unfortunately noise is difficult to get rid of, but taking multiple photographs over a period of time can help provide a more comprehensive picture.

2. Ensuring inversion about the mean

   Potential Challenge: A challenge particular to photonic quantum computing set-ups is the basis the detectors are in. Traditionally, a set of four detectors for a 2-qubit system would be in the states $00, 01, 10, 11$, respectively. However, the detectors may be in the diagonal basis or even a separate direction on the Bloch sphere altogether.

   Solution: Luckily, knowing the exact assignment of polarization states in the detectors is not completely necessary. Instead, as long as we confirm that one detector has significantly higher photon coincidences, we know that the marked state's amplitude was amplified. We can even confirm our results by removing the state marking and observing the outcomes at the director.

   Potential Challenge: Traditionally, in Grover's algorithm, the desired state is "marked" using an operator known as the oracle. For a 2-qubit system, this is most often implemented as a controlled-Z (CZ) gate. However, the diffusion operator also requires a CZ gate. Our lab may not have the resources to accomodate the beamsplitters and polarizing beamsplitters for two CZ implementations.

   Solution: We can manually mark the state by applying a $\pi$ phase shift to whatever our desired state is. This phase shift will effectively flip the amplitude of the desired state, allowing us to properly visualize it once we apply the diffusion operator.

# 5 Proposed Timeline:

## 5.1 Experimental Timeline

**Mid-September: Formal Project Proposal**

Task: Develop and submit a detailed proposal outlining the experimental objectives, hypotheses, and methodology. This proposal will include a literature review, a plan for building the experimental apparatus, and a plan for measuring the outcomes.

Goal:Ensure approval and receive feedback from Mr. Hannum.

**Mid-December: Interim Benchmark 1:** Experimental Set-Up

Task: Use the newly purchased single photon source, along with waveplates, phase-shifters, beamsplitters, polarizing beamsplitters, and detectors to build the interferometer.

Objective: Verify that the modes are aligned, perfectly overlapping at the second beamsplitter, and outputting from the polarizing beam splitter. Ensure equal photon coincidences at each detector.

Goal: Establish the framework for the project, ensure equipment works correctly

**Mid-March: Interim Benchmark 2:** Experimental Set-Up Pt 2 and Data Collection

Task: Finish data collection from initial interferometer set-up by rotating the HWP in increments of $\pi/4$ radians. Produce multiple photographs of spatial distribution at the detectors. Construct the diffusion operator.

Objective: Verify an oscillatory pattern in photon detections when HWP is rotated in increments. Ensure the spatial modes are bunched at the detector from the spatial profile in multiple photographs. Verify that the diffusion operator can reasonably be constructed (adjustment will come later).

Goal: Verify results and conduct data analysis, establish the second part of the project.

**End of May: Final Benchmark: Complete Experiment and Prepare for TjStar**

Task: Finalize results from pt 1 of the experiment. Test the diffusion operator and recalibrate the phase shifts and controlled-Z gate operation if necessary. Collect results of photon detections again.

Objective: Verify a spike in photon coincidences at one of the detectors when the desired state is marked, otherwise ensure the coincidences are even.

Goal: Wrap up research, begin drawing detailed conclusions in the context of photonics

## 5.2 Research Paper Timeline

**Mid-December: First Draft of Introduction and Background**

Task: Write the introduction, the motivation in context of broader societal impacts, as well as a brief background on photonic quantum computing. Explain the novelty of an indistinguishable, polarization-encoded 2-qubit system using the spatial modes of a single photon.

Goal: Complete the introduction, as well as an outline and general ideas for the remaining sections in the paper.

**Mid-March: First Draft of Methods and Preliminary Results**

Task: Explain the two goals of the project and the methods for each section. Include the photon coincidences data from Pt 1, as well as diagrams, photographs, and Perceval renderings for the circuits.

Goal: Ensure the methods are clearly described and the preliminary results are reported accurately. Start looking into possible journals/conferences to publish.

**End of May: Final Research Paper**

Task: Include the results for Pt 2 in the Methods section and revise the draft to ensure clarity and intellectual merit. Explicitly state how this research is different from others in the field.

Goal: Submit the final paper detailing novel contributions and broader impacts, also submit for publication.

# 6 Annotated Citations

1. Fiorentino, M., & Wong, F. (2004). Deterministic controlled-not gate for single-photon two-qubit quantum logic. Physical Review Letters, 93(7). https://doi.org/10.1103/physrevlett.93.070502

   - Fiorentino and Wong (2004) developed a deterministic controlled-NOT (CNOT) gate for quantum logic using single photons. Their approach leverages linear optical elements, specifically a polarization Sagnac interferometer combined with a dove prism, to control the interaction between two qubits encoded within a single photon—one in polarization and the other in spatial (momentum) mode. This setup allows for efficient entanglement between these qubits without requiring active stabilization, marking a significant advancement in scalable quantum information processing.

2. Kwiat, P.G., Mitchell, J.R., Schwindt, P.D.D., & White, A.G. (1999). Grover's search algorithm: an optical approach. Journal of Modern Optics, 47(2/3), 257-266. https://research.physics.illinois.edu/QI/Photonics/papers/My%20Collection.Data/PDF/Grover's%20search%20algorithm%20An%20optical%20approach.pdf

   - The study presents an all-optical approach where qubits are encoded in different degrees of freedom, such as photon polarization and spatial modes. They reduced the number of optical elements required for the algorithm from 24 to 12, demonstrating the potential of optical systems for quantum computations. Their setup allows for efficient querying of a small database with high accuracy and highlights the possibility of scaling the approach to larger systems by adding more spatial modes. This work showcases the potential of quantum optical systems in performing computational tasks like database searches