Research Proposal

High-Resolution Turbulence Modeling Using the HHL Algorithm for Poisson Navier-Stokes Equations

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1 Introduction:

The proposed research aims to investigate the application of quantum computing techniques, specifically the HHL (Harrow-Hassidim-Lloyd) algorithm, to enhance the solutions of the Poisson equation in computational fluid dynamics (CFD). The Poisson equation is a fundamental partial differential equation that describes potential fields in various physical systems, including fluid flow. Classical methods for solving the Poisson equation can be computationally expensive, especially for high-dimensional problems. This research seeks to leverage quantum algorithms to reduce computational time and improve resolution for turbulence modeling.

1.1 Intellectual Merit

The intellectual merit of this research lies in its capacity to bridge quantum computing and classical CFD methodologies. Previous studies have highlighted the limitations of classical approaches when scaling up to complex fluid systems. By employing the HHL algorithm, which offers exponential speedup for solving linear systems of equations, this project aims to provide empirical evidence of the algorithm's effectiveness in practical CFD applications. Moreover, this research could offer insights into the broader applicability of quantum algorithms in other areas of physics and engineering, thereby advancing our understanding of quantum-enhanced computing capabilities.

1.2 Broader Impacts

The broader impacts of this research are significant, as they align with the growing interest in integrating quantum computing into fluid dynamics. If successful, the findings could contribute to the relatively small pool of CFD techniques that accelerate and provide high-resolution simulation data for anything from aerospace to atmospheric applications. This research leaps into a relatively new QC technique, and will hopefully become a stepping-stone for

2 Specific Project Goals and Measurements:

1. **HHL as a viable method for high-resolution turbulence modeling**
   The primary goal is to implement the HHL algorithm to solve the Poisson equation within a CFD framework. We will measure the accuracy of the quantum solution compared to classical numerical methods. Success will be indicated by demonstrating that the quantum solution converges to the expected results with significantly reduced computational time.

2. **Explore supplementary methods/algorithms to enhance HHL algorithm**
   By utilizing the HHL algorithm, it'd be interesting to see how other methods can be used to decrease computational costs and increase processing speed. For instance, by running the HHL algorithm in a partitioned mesh while using Classical approaches to larger-scale turbulence modeling, such as Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS), we can more effectively compute models with localized resolution.

3 Specific Responsibilities of Group Members:

Taiki Yamauchi
Responsibility: Developing and optimizing the quantum circuit for solving the Poisson equation using the HHL algorithm.

Tasks: Design the quantum circuit, ensuring that it accurately represents the problem and can handle fluid dynamics equations like the Poisson and Navier-Stokes. Optimize the circuit to reduce complexity, such as minimizing gate count and depth.

Ryeen Afkhami
Responsibility: Setting up the classical computational fluid dynamics (CFD) framework and comparing results with the quantum circuit.

Tasks: Develop the classical CFD model, focusing on solving the Navier-Stokes and Poisson equations.

Validate the accuracy of classical results and benchmark them against quantum results.

Handle data collection, analysis, and interpretation, including performance comparisons between quantum and classical methods.

Prepare visualization and documentation of findings, ensuring both quantum and classical results are clearly presented.

4 Potential Pitfalls and Alternative Strategies:

1. Validation of HHL Algorithm in Turbulence Modeling:

    Potential Challenge: Translation of the turbulence equations into a form suitable for the HHL algorithm, especially regarding discretization methods. Inadequate discretization could lead to errors in the quantum solution.

    Solution: Conduct preliminary tests using simplified versions of turbulence equations to refine our discretization techniques before applying them to more complex scenarios.

    Potential Challenge: The ability to execute and test our algorithm due to QC availability.

    Solution: Implement classical approaches to turbulence modeling simultaneously, ensuring we can gather data regardless of quantum computer availability.

2. Possible complexity of the Quantum Circuit and Algorithm

    Potential Challenge: Circuit Complexity with Increased Dimensionality: As our simulated objects become more geometrically complex, the circuit required to run the HHL can grow significantly.

    Solution: We can start initially modeling 1D turbulence, then lay that as a foundation for the convergent-divergent (de Laval) nozzle. From there, we can use certain techniques such as cross-sectional analysis to reduce 3D contours to 2D splines.

    Potential Challenge: As the size of the fluid dynamics problem increases (e.g., larger grids for turbulence modeling), the number of qubits and quantum gates required to accurately represent the system can become impractically large. This can lead to excessive circuit depth, increasing the likelihood of errors and noise affecting the computation.

    Solution: We can reduce the number of gates and the circuit's depth by rearranging operations through gate commutation and eliminating unnecessary components via circuit simplification. We can also explore Classiq, which can turn Python into a circuit.

5 Proposed Timeline:

5.1 Experimental Timeline

Mid-September: Formal Project Proposal

Task: Draft and submit a proposal highlighting the method, hypothesis, and aim of the proposed research. The proposal will contain annotated references of relevant literature, address any potential limitations of the study, and provide a background of past research in the field.

Goal: Receive approval and feedback for the project and coordinate with potential mentors.

Mid-December: Interim Benchmark 1: Algorithm Implementation

Task: Implement the HHL algorithm and set up the CFD environment for initial testing focused on 1D turbulence models.

Objective: Verify the algorithm's functionality within the turbulence modeling framework.

Goal: Develop a strong foundation for the HHL framework. Present preliminary findings regarding the feasibility of using the HHL algorithm in turbulence modeling.

Mid-March: Interim Benchmark 2: Data Collection and Analysis

Task: Conduct experiments comparing quantum and classical solutions to turbulence modeling scenarios.

Objective: Collect data on accuracy and computational efficiency. Look into any simulation inefficiencies and bugs.

Goal: Analyze results and draw preliminary conclusions on the effectiveness of HHL algorithm.

End of May: Final Benchmark: Complete Experiment and Prepare for TjStar

Task: Finalize data collection. Work on broader applications, comparing simplified 3D models to classical CFD.

Objective: To confirm the effectiveness of an HHL in high-resolution, macroscopic turbulence modeling.

Goal: Provide empirical evidence for the advantages of quantum enhancement in turbulence modeling. Perhaps solidify potential ways to reduce noise, and increase accuracy. Find ways to make HHL a more implementable QCFD framework.

6 Annotated Citations

1. S. Bharadwaj1 and K. Sreenivasan (2023). ”Hybrid quantum algorithms for flow problems.” 1-21
    - Bharadwaj1 and Sreenivasan of NYU sought to develop a Quantum flow simulator using QC. More specifically, they developed a framework which classically implemented a hybrid HHL and Quantum Linear Systems Algorithms (QLSA) to model and analyze simple turbulence models. This work is an important and successful attempt at a hybrid GCFD program reared toward a wide range of uses.
2. K. Fukagata (2022).”Towards quantum computing of turbulence.” Nature Computational Science, 2, 68–69
    - Fukagata highlights the implications of QC to minimize the computational cost for increasing Reynold turbulence. Through Schmidt Decomposition of the classical velocity, Fukagata shows that can utilize quantum entanglement to modify the classical model. This work is an important analysis of the implications of translating classical turbulence equations for QC and opens a discussion into possible methodologies for QCFD.