Quantum Harmonic Oscillator Model of Stock Returns

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Research

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Stock Returns

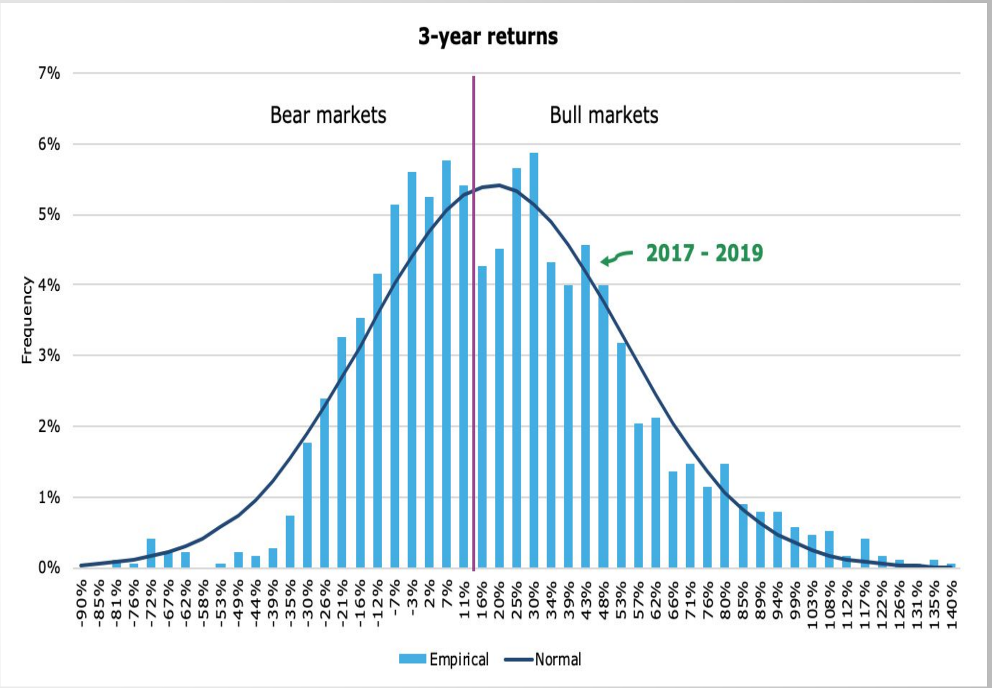
- Simple Returns:

  $r_t = (P_t - P_{t-1})/(P_{t-1})$, where P_t is the stock price at time t. r_t is the return. Good for short turn percentage calculations

- Log Returns:

  $r_t = \ln(P_t/P_{t-1})$. Useful for modeling continuous distributions. r_t is the log return. Good for long term financial modeling.

  Takes into account the compounding effect of stock returns.



Example of a stock return distribution.

Modeling Stock Returns

Classical Models:

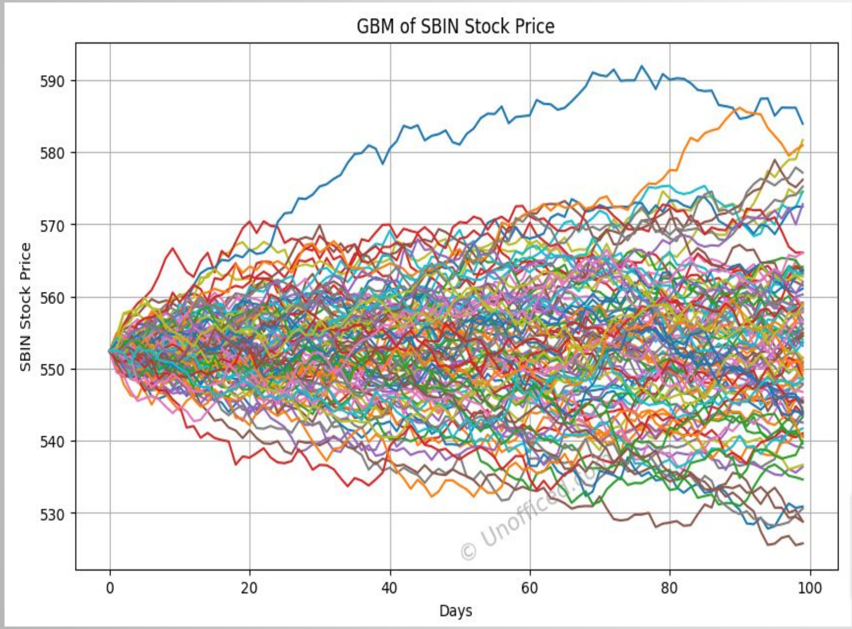
- Statistics/Math models
- Examples include brownian motion, black-scholes model, and monte carlo simulations.
- May not be as effective because empirical evidence illustrates that the distribution of stock returns has non-Gaussian properties including negative skewness
- Simpler and more widely used currently

Quantum Models:

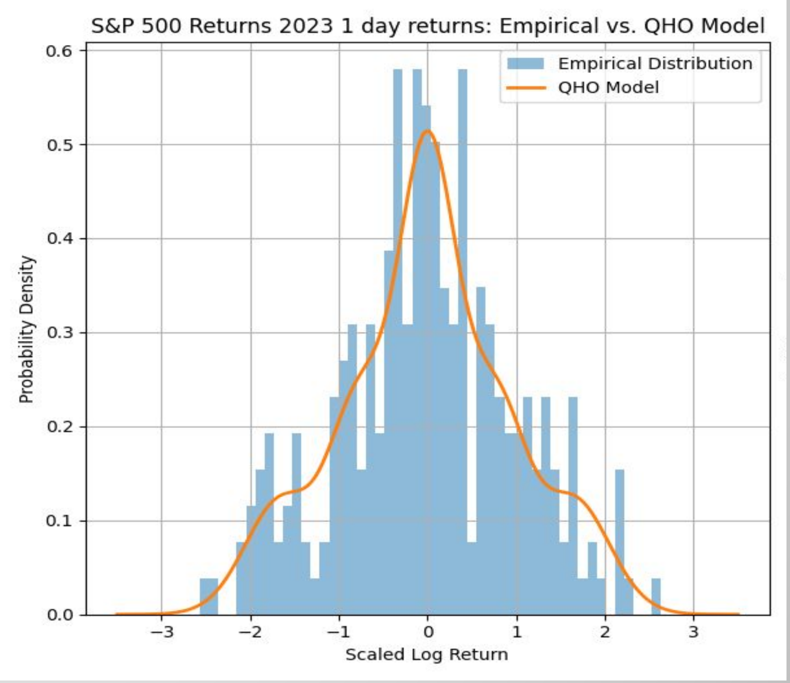
- Uses quantum superposition to represent multiple price paths simultaneously.
- Helps model market uncertainty beyond classical stochastic processes.
- Treats market evolution as a wavefunction, allowing a continuous price probability distribution.
- Currently not widely used due to limited quantum technology
- Quantum computing has the potential to advance quantum models, which are faster and can better account for non normal fluctuation of stocks.

Modeling Long Term Stock Return Distributions

Classical Model Example:



Quantum Model Example:



Research Question

Can the Quantum Harmonic Oscillator be used to accurately model future long term stock return distributions by time evolution?

Methods

- Import data for S&P 500 and Nvidia from yahoo finance packages in python
- Define the Quantum Harmonic Oscillator
- Fit a superposition of the eigenfunctions of the quantum harmonic oscillator to the different stock return distributions
- Time evolve the quantum harmonic oscillator
- Create graphs of the the quantum model and graph it with the empirical distributions
- Graph the time evolved model
- Compare time evolved model to actual distribution of the “future” empirical distribution.

Some python packages I used:

```
import pandas as pd
import yfinance as yf
import numpy as np
import matplotlib.pyplot as plt
from scipy.special import
eval_hermite
from scipy.integrate import
simps
from scipy.optimize import
curve_fit
import math
```

Quantum Harmonic Oscillator Model & Schrodinger Equation

- The quantum harmonic oscillator is a system where a particle experiences a restoring force proportional to its displacement.
- Potential energy given by V(x)
- Solving the schrodinger equation by substituting V(x) gives the energies of the quantum harmonic oscillator
- These can be used to find the eigenfunctions.

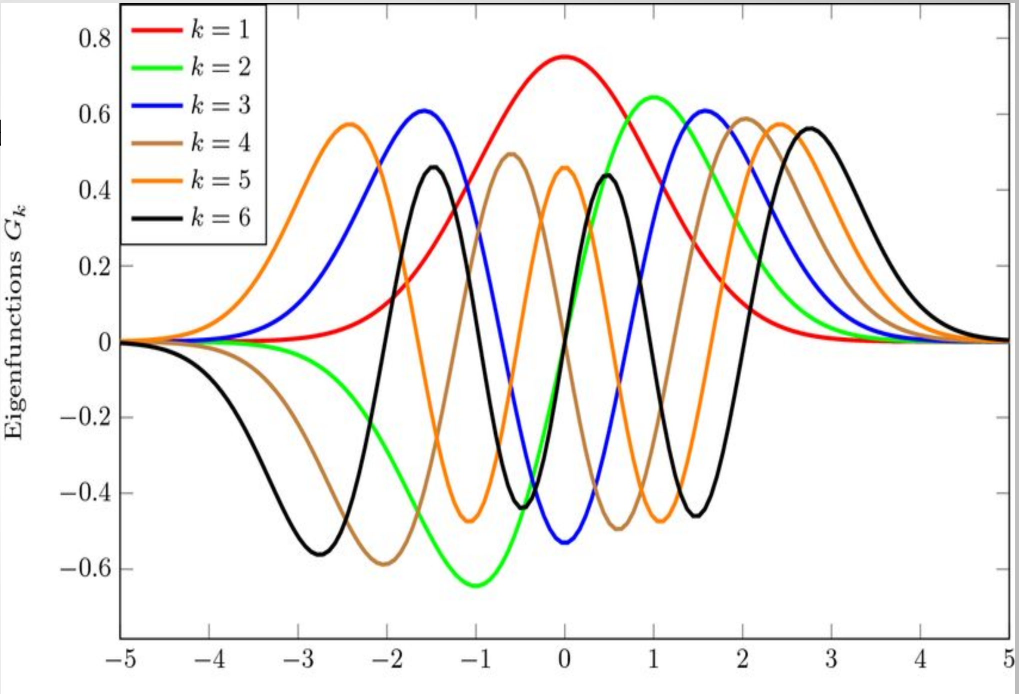
$$i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

$$V(x) = \frac{1}{2}m\omega^2x^2$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \quad n = 0, 1, 2, \ldots$$

Quantum Harmonic Oscillator Eigenfunctions

- Eigenfunctions
  - Functions that become scaled by a factor w
    acted upon by an operator
  - ef(x) = H f(x)
- Quantum Harmonic Oscillator Eigenfunctions
- $$\Psi_n(x) = N_n H_n\left(\frac{x}{\sqrt{2}\,l}\right) e^{-\frac{x^2}{2l^2}}$$
- Can use superposition of the square of the eigenfunctions to model stock distributions
  - For example, a distribution may by 0.7 times the first function plus 0.2 times the second function plus 0.1 times the third function

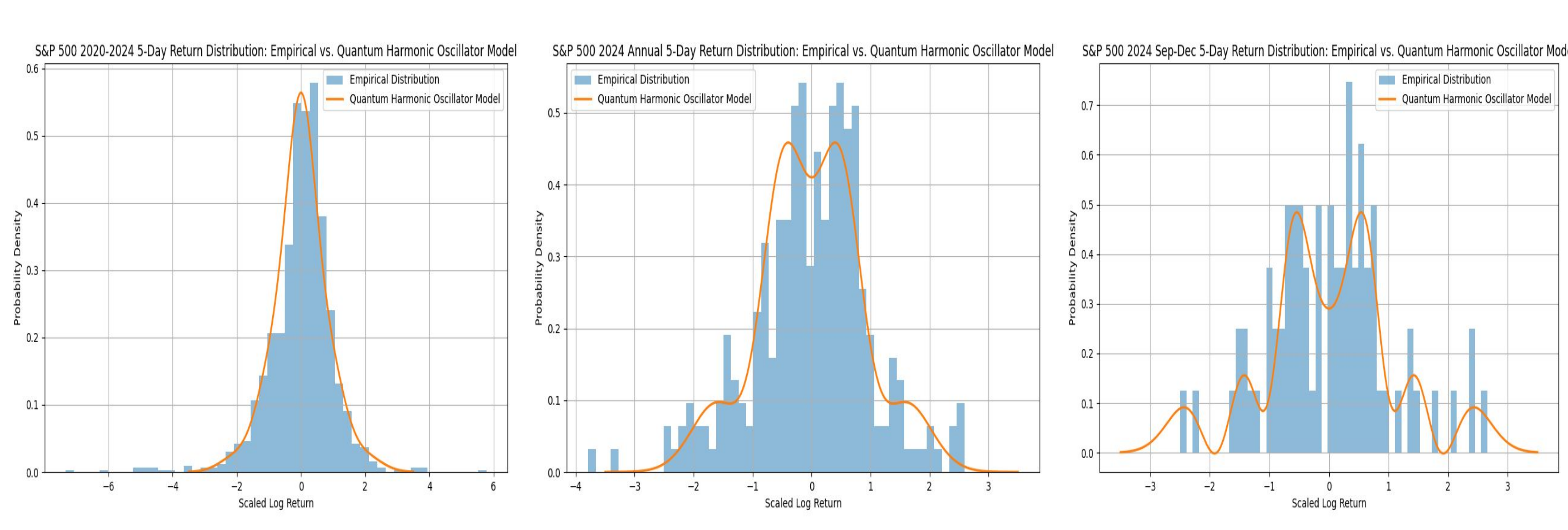


Quantum Harmonic Oscillator Time Evolution

- The eigenfunction superposition, or wave, oscillates as time passes by
  - Original superposition is symmetric but after time evolution the eigenfunction may not be symmetric anymore
- If the initial superposition function is defined by $\Psi(x,0) = \sum_n c_n \Psi_n(x)$
- Then the time evolution is $\Psi(x,t) = \sum_n c_n \Psi_n(x) e^{-iE_n t/\hbar}$
- Phase factor $e^{-iE_n t/\hbar}$
- Time evolving a quantum harmonic oscillator can show the continuous price transitions of stocks and can be used to predict price fluctuations over long periods.
- Quantum Model represents a probability distribution of stock prices

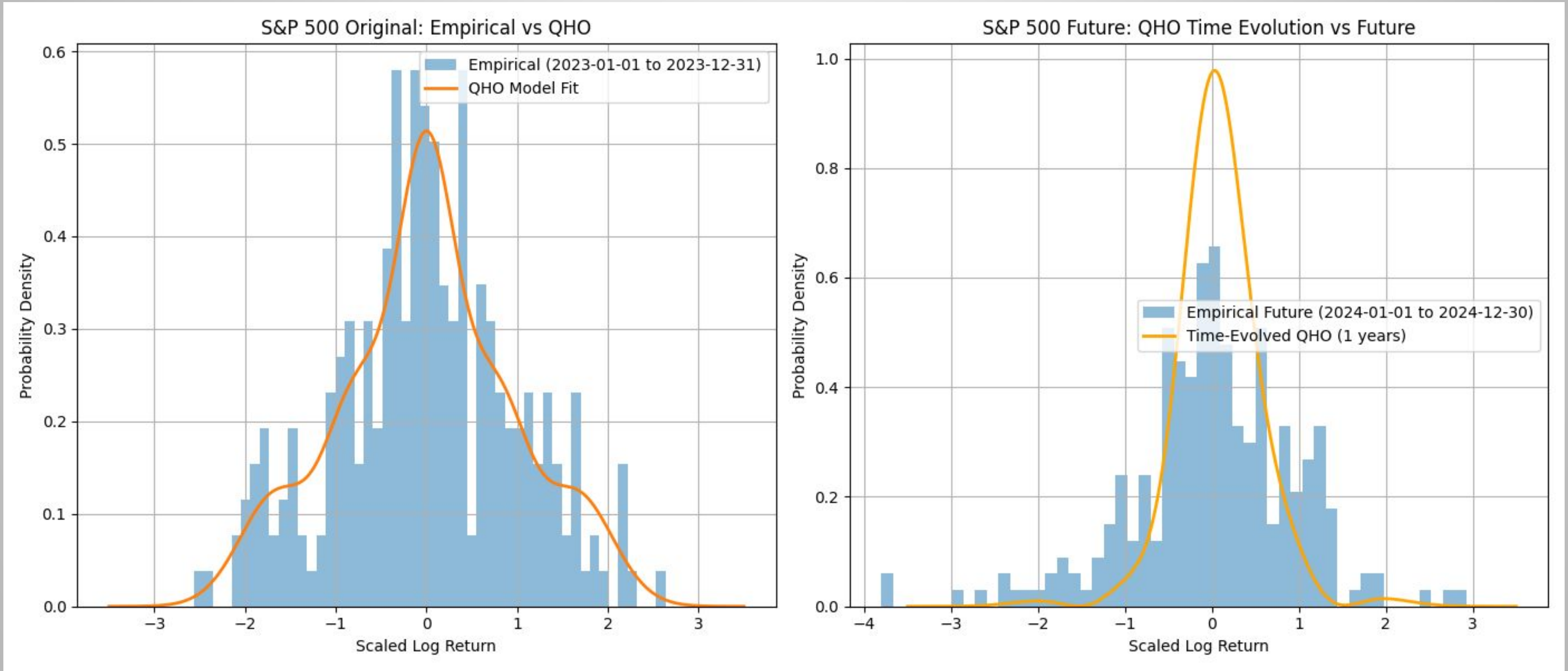
Results

Modeling past stock returns for S&P 500, four months, one year, and five years trading duration graphs for 5 day trading periods in 2024.

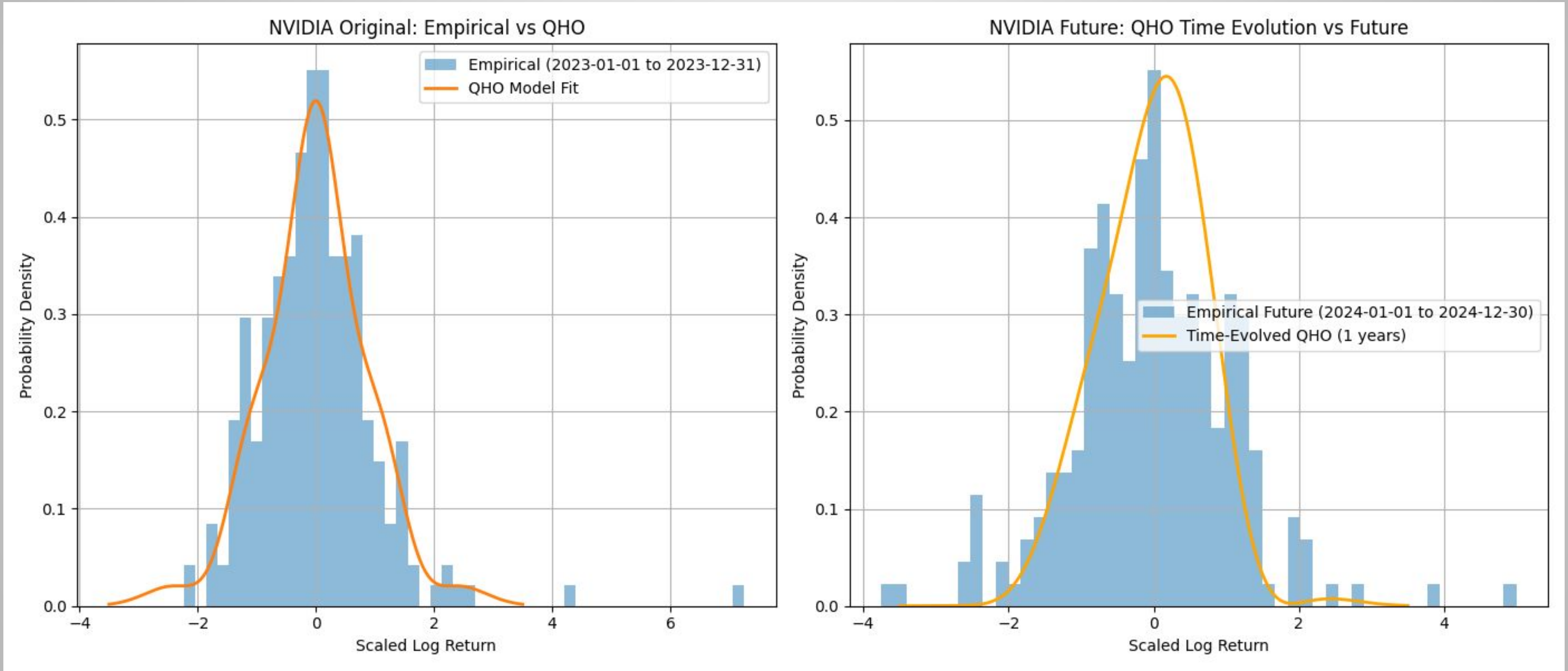


I decided to focus on different trading periods for a one year duration.

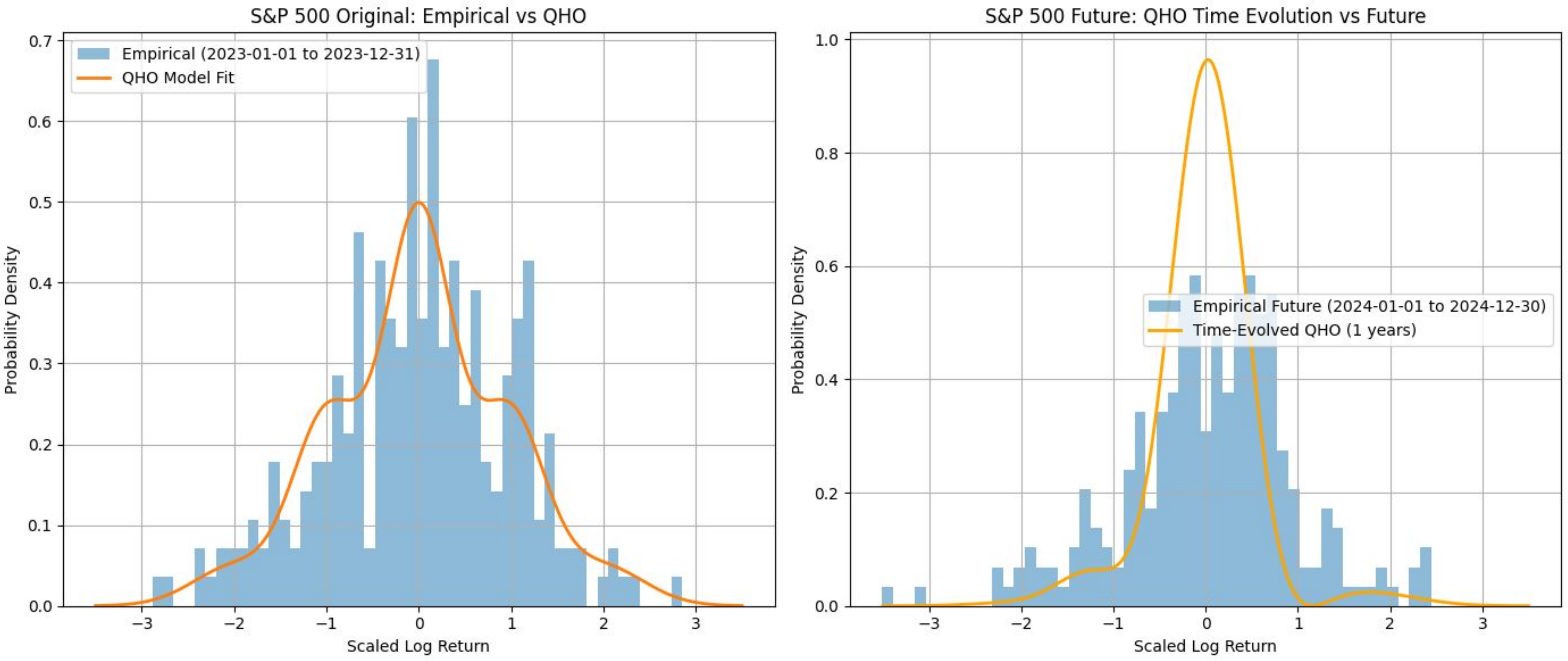
S&P 500 1 Day Returns 2023, 2024



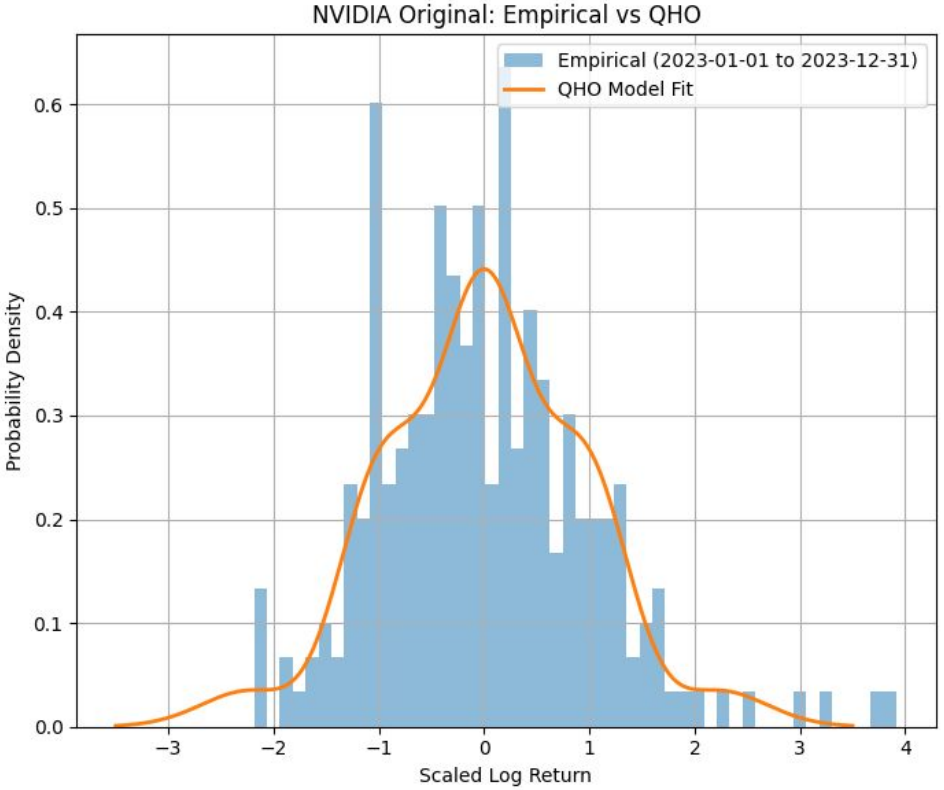
NVIDIA 1 Day Returns 2023, 2024

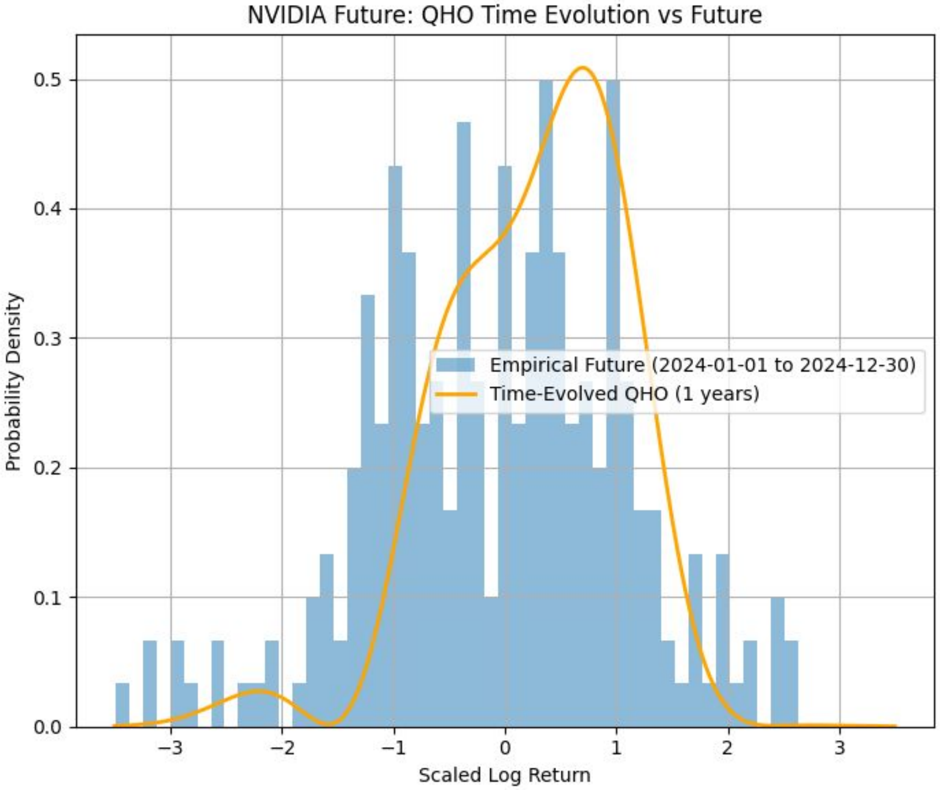


S&P 500 5 Day Returns 2023, 2024

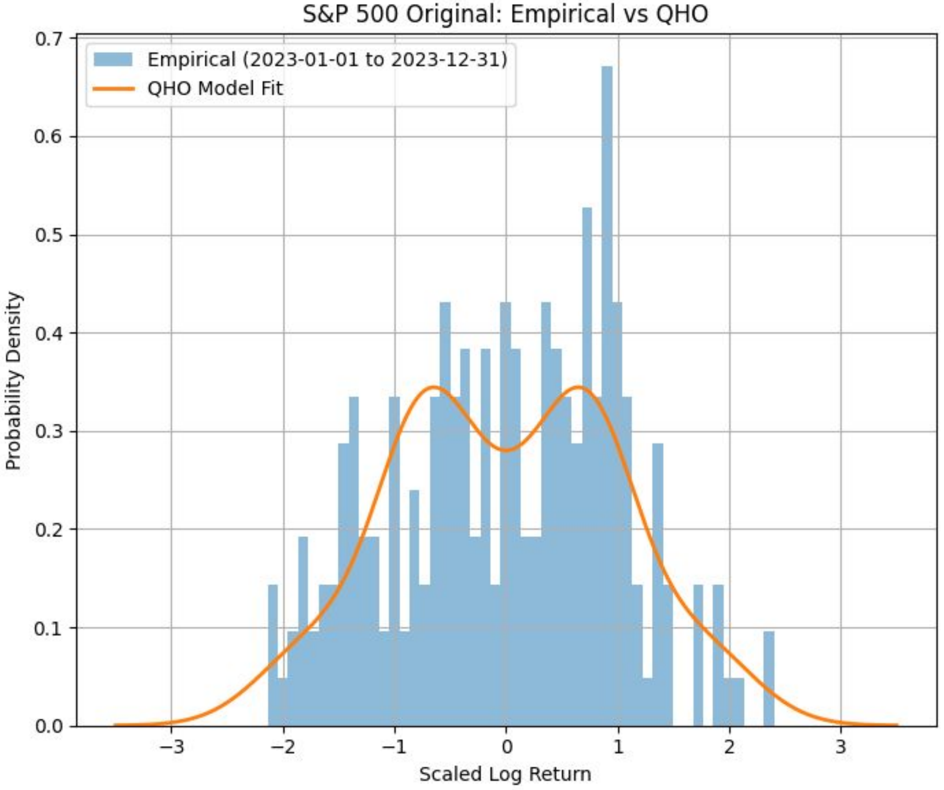


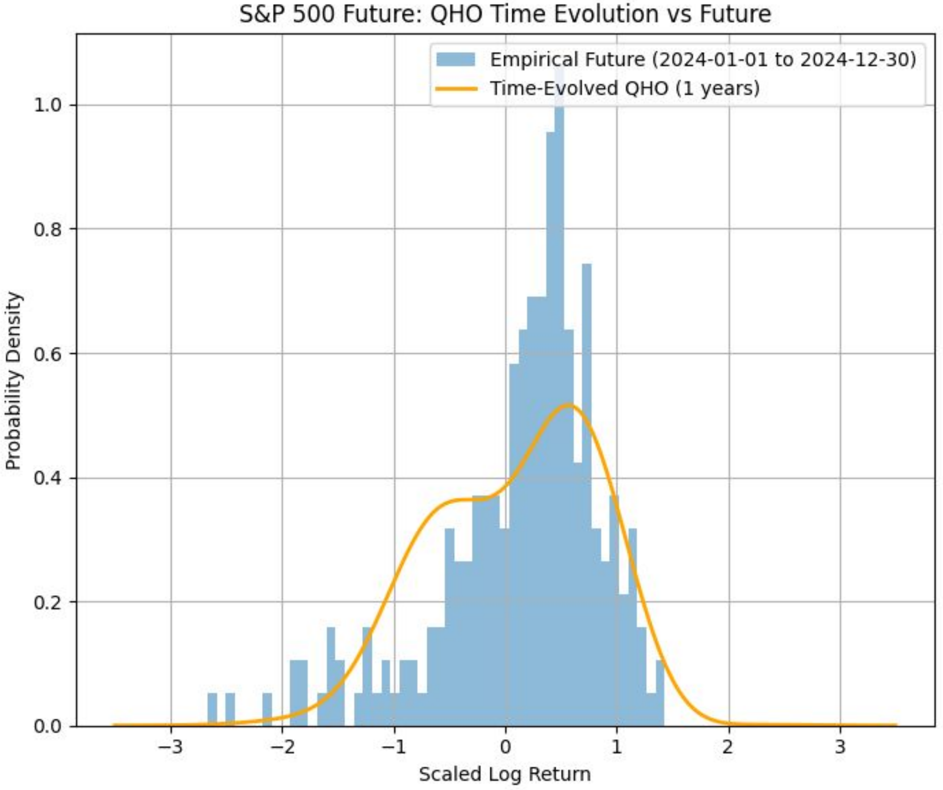
NVIDIA 500 5 Day Returns 2023, 2024



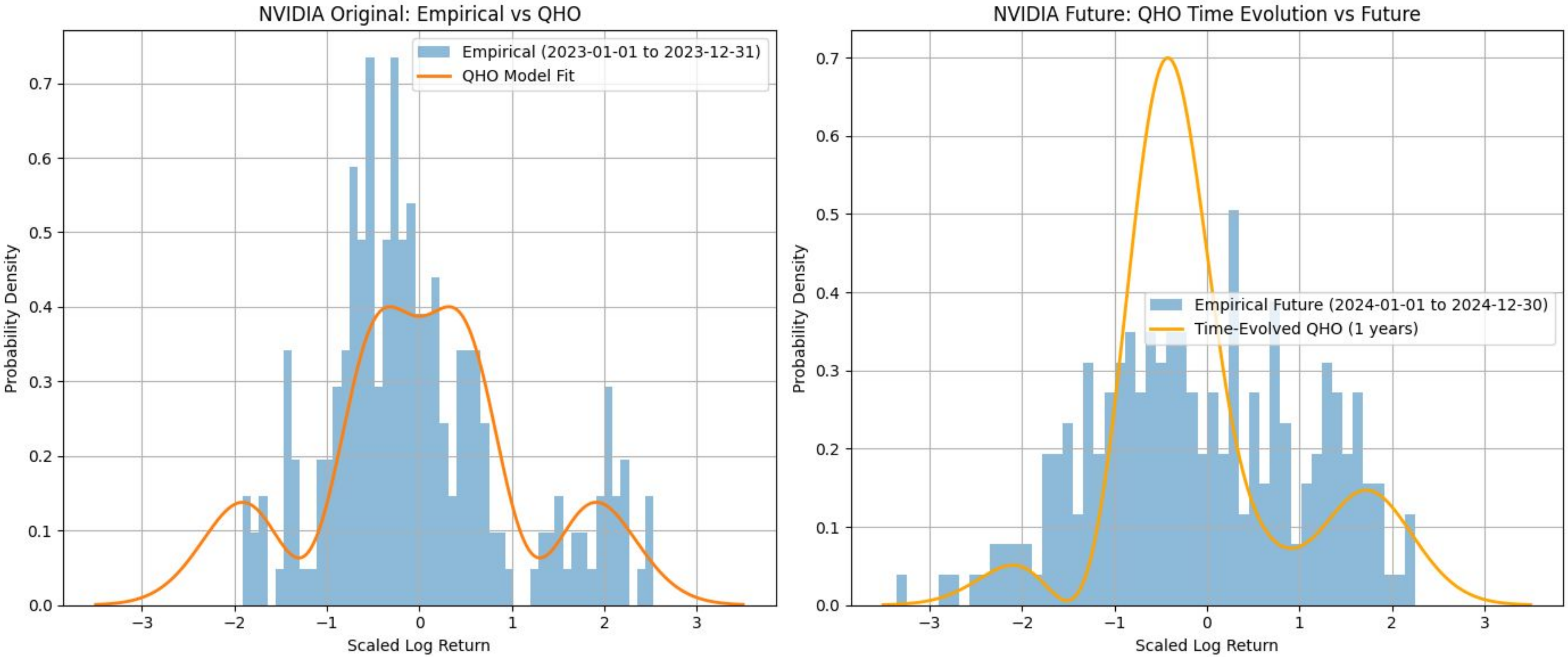


S&P 500 20 Day Returns 2023, 2024





NVIDIA 500 20 Day Returns 2023, 2024



Discussion

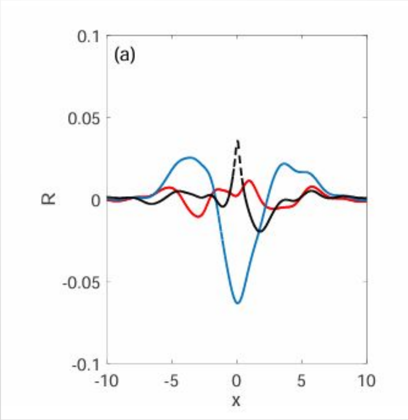
- The Quantum Harmonic Oscillator model mostly aligned with empirical distributions and results of other papers, which made sense.
- The time evolved model for the most part, seemed to match the “future” empirical distribution.
- My project could have been better if I had ran statistical tests to test how much error the time evolved model had.
- I could have seen how much the curve of the graph deviated from each of the data points of the empirical distribution and average the deviations to get the errors.

Discussion

- The main extension of my project from previous papers was adding a time evolved model.
- Throughout my project, I had the most difficulty assessing the accuracy of my time evolved model due to not having previous papers to compare my results with, so my interpretations were from the graphs.
- Another reason this was problematic, was because I was not able to code statistical tests to accurately measure the error of my time evolved model.

Discussion

- Future directions
  - Test time evolving the quantum harmonic oscillator model for other durations. E.x. multiple years, or a few months.
  - Create statistical tests to determine the magnitude of the error
  - Compare time evolved quantum model with other long term classical predictors of stock distributions



Conclusion

- Quantum Harmonic Oscillator is useful for modeling long term distributions of stocks of the past
- Quantum Harmonic Oscillator may not be great for predicting future stock returns
- Has potential for becoming more useful for predicting market shifts in the future as quantum mechanics and quantum computing become more developed

Thank You
Questions?