Quantum Harmonic Oscillator Model of Stock Return Distributions

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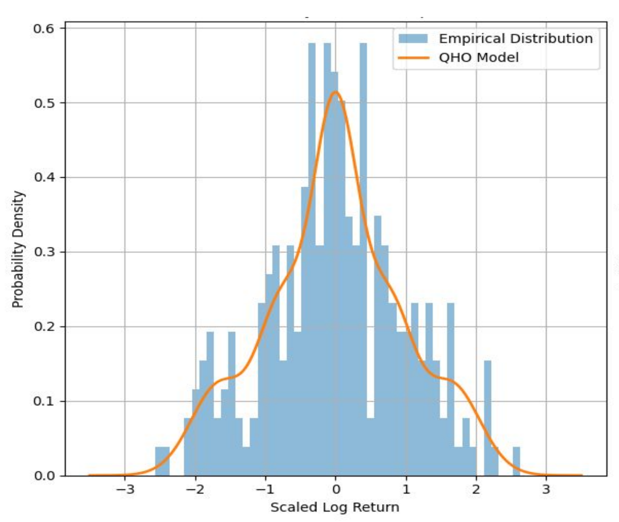
Introduction

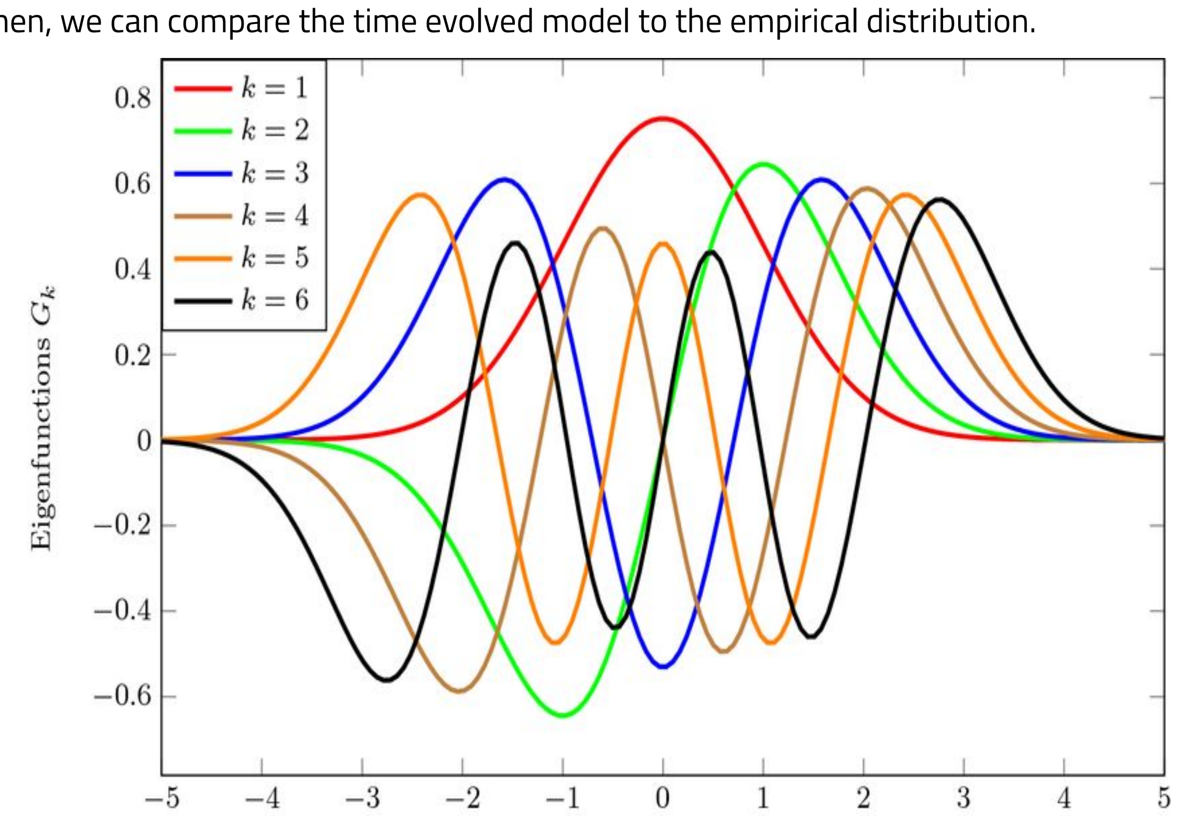
This project uses the quantum harmonic oscillator to model return distributions. Quantum harmonic oscillators have a restoring force that pulls the system back to equilibrium. In quantum models, energy levels take on specific values, which can better capture the discrete nature of transactions and price changes in the stock market. Stock prices change in discrete steps due to individual transactions. Each trade can be seen as moving the stock price from one “energy level” to another. Quantum oscillations are described by wave functions which can give probability distributions to measure for a particular position. The added variability in quantum oscillations better model volatility and better non-normal distributions of stock price. Modeling the stock market using quantum oscillations can be useful for companies and businesses at large for their investments. This includes allowing for better investing such as more accurate financial analysis that can better predict market interactions and changes. This could lead to more effective use of financial tools and better risk management in the long term. Quantum oscillations for stock markets can also lead to interdisciplinary applications such as combining physics with data and economics.



Example of a quantum harmonic oscillator model paired with an empirical distribution of stock data from past distributions.

Methodology

Download data for S&P 500 and Nvidia from yahoo finance packages in python
Define the Quantum Harmonic Oscillator using potential energy given by V(x).
Solving the Schrodinger equation by substituting V(x) gives the eigenenergies of the quantum harmonic oscillator.
These can be used to find the eigenfunctions, which are functions that become scaled by a factor when acted upon by an operator.
We can fit a superposition of the eigenfunctions of the quantum harmonic oscillator to the different stock return distributions.
We then time evolve the quantum harmonic oscillator. The eigenfunction superposition oscillates as time passes by. While the
original superposition is symmetric, after a time evolution, the model may not be symmetric anymore, which can capture non-normal aspects of stock returns.
We then create graphs of the the quantum model and graph it with the empirical distributions. We then graph the time evolved model.
Then, we can compare the time evolved model to the empirical distribution.



A visual representation of the eigenfunctions of the quantum harmonic oscillator from the ground state to the 5th excited state.

$$i\hbar\frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$$

Schrodinger Equation

$$V(x) = \frac{1}{2}m\omega^2x^2$$

Potential function V(x)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \quad n = 0, 1, 2, \ldots$$

Eigenenergies

$$\Psi_n(x) = N_n H_n\left(\frac{x}{\sqrt{2}l}\right)e^{-\frac{x^2}{2l^2}}$$

Eigenfunctions

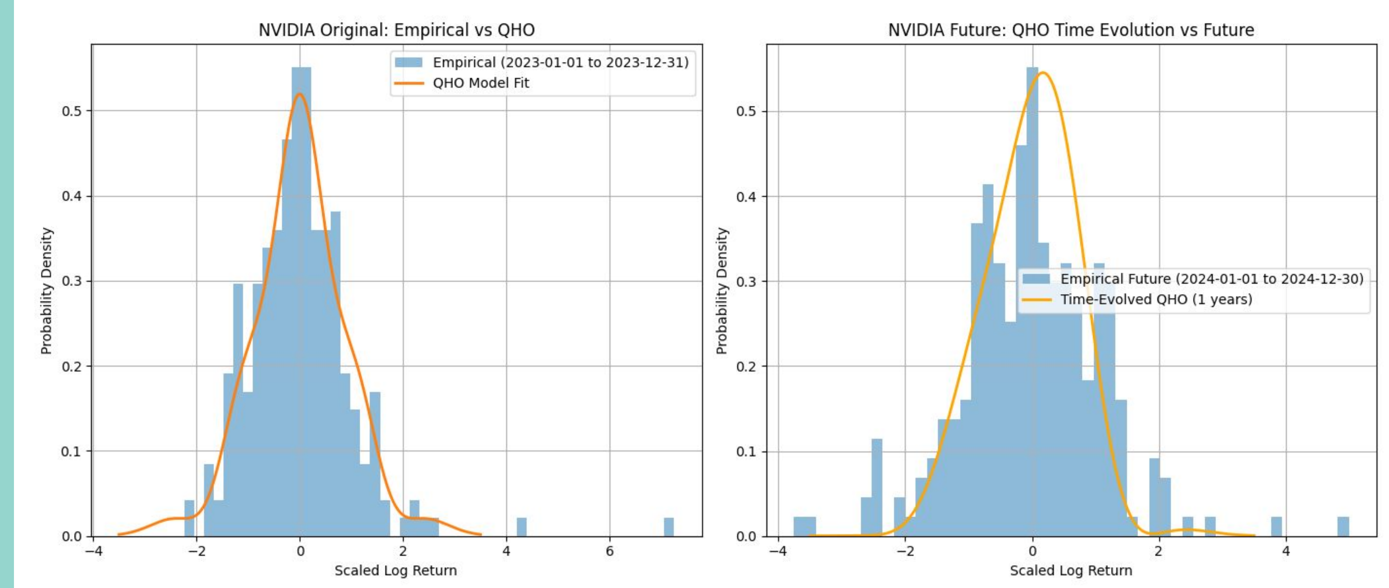
$$\Psi(x,0) = \sum_n c_n\Psi_n(x)$$

Wavefunction at time 0

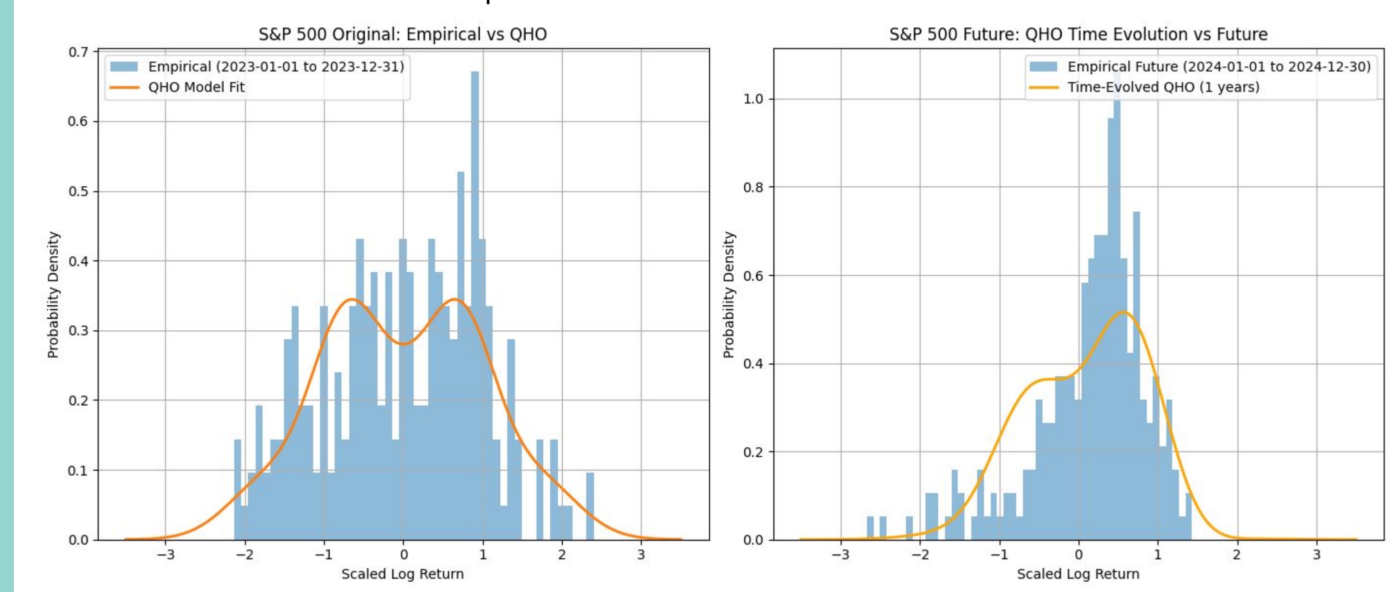
$$\Psi(x,t) = \sum_n c_n\Psi_n(x)e^{-iE_nt/\hbar}$$

Wavefunction at time t, evolved by applying phase factor exp[(-i*nth Eigenenergy*time)/(h bar constant)]

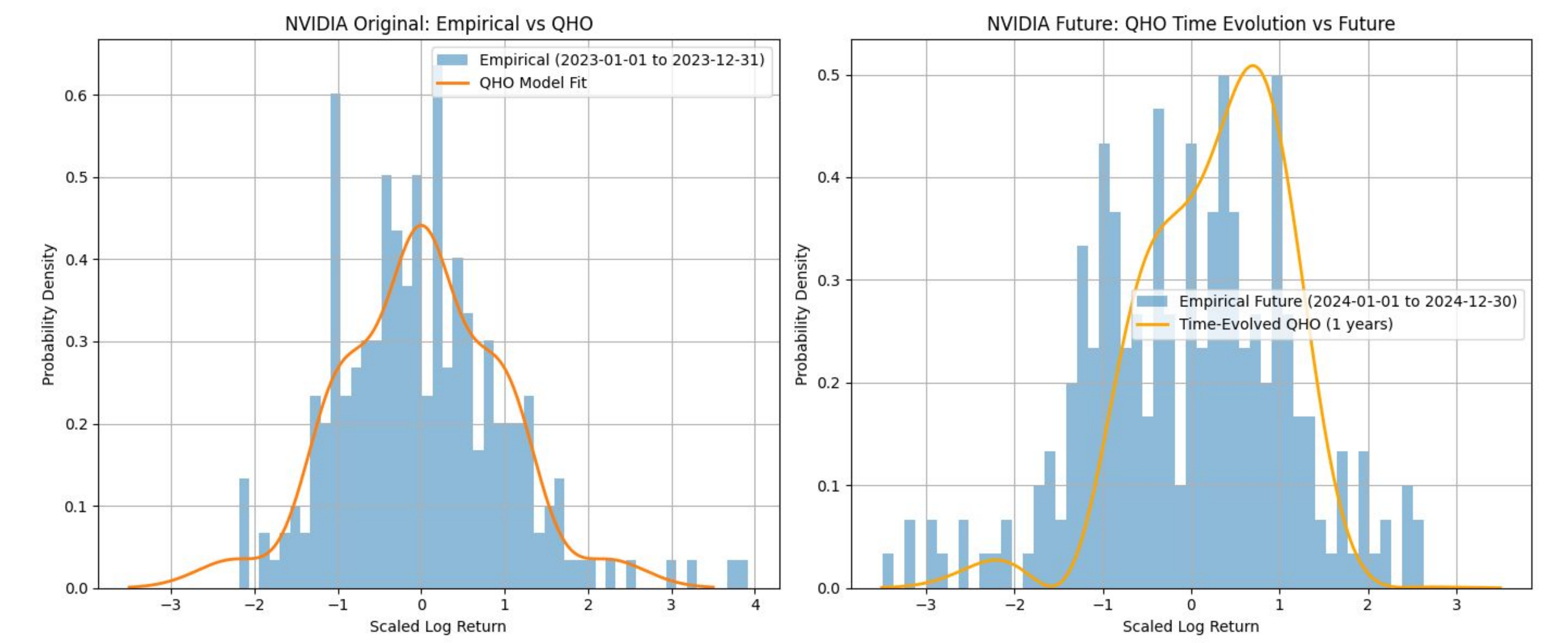
Results



Result for 1 day stock returns. The graph on the left shows the original quantum harmonic oscillator model fitted to the 2023 stock distributions of NVIDIA. The graph on the right shows the time evolved model compare to the ‘future’ 2024 stock distributions of NVIDIA.



Result for 20 day stock returns. The graph on the left shows the original quantum harmonic oscillator model fitted to the 2023 stock distributions of S&P 500. The graph on the right shows the time evolved model compare to the ‘future’ 2024 stock distributions of S&P 500.

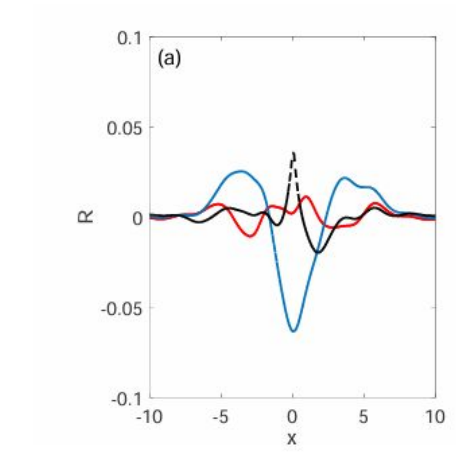


Result for 5 day stock returns. The graph on the left shows the original quantum harmonic oscillator model fitted to the 2023 stock distributions of NVIDIA. The graph on the right shows the time evolved model compare to the ‘future’ 2024 stock distributions of NVIDIA.

Overall, the results show that the Quantum Harmonic Oscillator model is suitable for capturing the general distribution of stocks of future dates based on past stock data for a duration of one year. There does not seem to be a clear difference in how accurate the model is for different trading periods.

Discussion

The Quantum Harmonic Oscillator model mostly aligned with empirical distributions and results of previous papers. The time evolved model for the most part, seemed to match the future empirical distribution. My project could have been better if I had ran statistical tests to test exactly how much error the time evolved model had. The main extension of my project from previous papers was adding a time evolved model. The time evolved model was able to catch non-normal fluctuations in the future stock price. Throughout my project, I had the most difficulty assessing the accuracy of my time evolved model due to not having previous papers to compare my results with, so my interpretations were from the graphs. Another reason this was problematic, was because I was not able to code statistical tests to detect the error of my time evolved model. For future directions, scientists should test time evolving the quantum harmonic oscillator model for other time durations, i.e. multiple years, or a few months. They could also test the model for penny stocks They could actually create statistical tests to determine the magnitude of the error and compare time evolved quantum model with other classical predictors of stock distributions.



Example of comparing the errors multiple different stock return models with one another

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Acknowledgements

Thank you to Mr. Hannum for being a chill lab director.